

## THERMAL EFFICIENCY OF STIRLING ENGINE

### 1. INTRODUCTION

Stirling engine, invented and patented by Robert Stirling in 1816, is a machine which converts thermal energy into mechanical one. Its characteristic feature that distinguishes it from any other heat engine is that it contains the fixed amount of gas (hydrogen, helium, air). There is no gaseous mixture of air or liquid fuel which is burnt cyclically during its operation but heat is continuously supplied to it from the outside (e.g. heat from the combustion of solid, liquid or gas fuel, the heat provided by the solar collector, the heat of fermentation of manure). The working heated medium flows between two cylinders with pistons, called pumping and working or warm and cold one. Thus there is no exhaust valves for gases that are released under high pressure, as well as the detonation combustion of the fuel composition.

Table 1. Advantages and disadvantages of the Stirling engine

Advantages	Disadvantages
<ul style="list-style-type: none"> <li>- very low noise;</li> <li>- a very low level of harmful emissions when used in a large variety of heat sources;</li> <li>- high thermal efficiency;</li> <li>- no ignition system (relatively expensive and highly complex);</li> <li>- acceptability of loose fitting hot piston for pumping working medium;</li> <li>- work (practically without any structural changes) as a heat pump or refrigerating equipment</li> </ul>	<ul style="list-style-type: none"> <li>- difficulty in obtaining smooth speed changes;</li> <li>- a relatively large size in relation to the engine power obtained;</li> <li>- the need for intense cooling of the working medium flowing into the chamber above the piston and a relatively high self-weight when the working fluid pressure exceeds 2 MPa</li> </ul>

Considering the advantages of the Stirling engine it is applicable in the following areas

energy :

- stationary and portable power generators ,
- systems - combined production of heat and electricity ,
- processing systems for solar energy

automotive industry :

- main drive motor vehicles,
- hybrid powertrain vehicles

maritime industry :

- drive submarines
  
- auxiliary power units and generators,
- heat treatment systems exhaust gas marine engines ,
- generators for charging sailing boats,
- drive a small robot submarine

space industry :

- auxiliary energy sources in power systems of spacecraft ,

medicine:

- artificial heart pump ,

refrigeration, air conditioning and cryogenic

heat pumps and water pumps,

model-making :

- drive miniature models of vehicles,
- drive miniature models of vessels

## 2. CONSTRUCTION AND OPERATION OF STIRLING ENGINE

Figure 1 shows a view of the Stirling engine purchased at PHYWE. In the basic configuration (Fig. 1) engine consists of two cylinders (hot and cold). Cylinder warm (horizontal) is continuously heated using spirit burner and cold cylinder (vertical) is cooled by ambient air. Both cylinder chambers are connected to each other in such a way that gas flows freely between them (through a vertical glass tube - Fig. 1). Visible at the end of the tube radiator facilitates heat dissipation to the environment. The pistons are connected to the crankshaft in such a way that the warm (untight) piston in the cylinder ahead of the movement of the piston in the cold cylinder (sealed) by an angle of  $90^\circ$

In order to sustain the motion of the pistons when the engine does not generate the driving force the connecting rod is connected to the flywheel (Fig. 2)

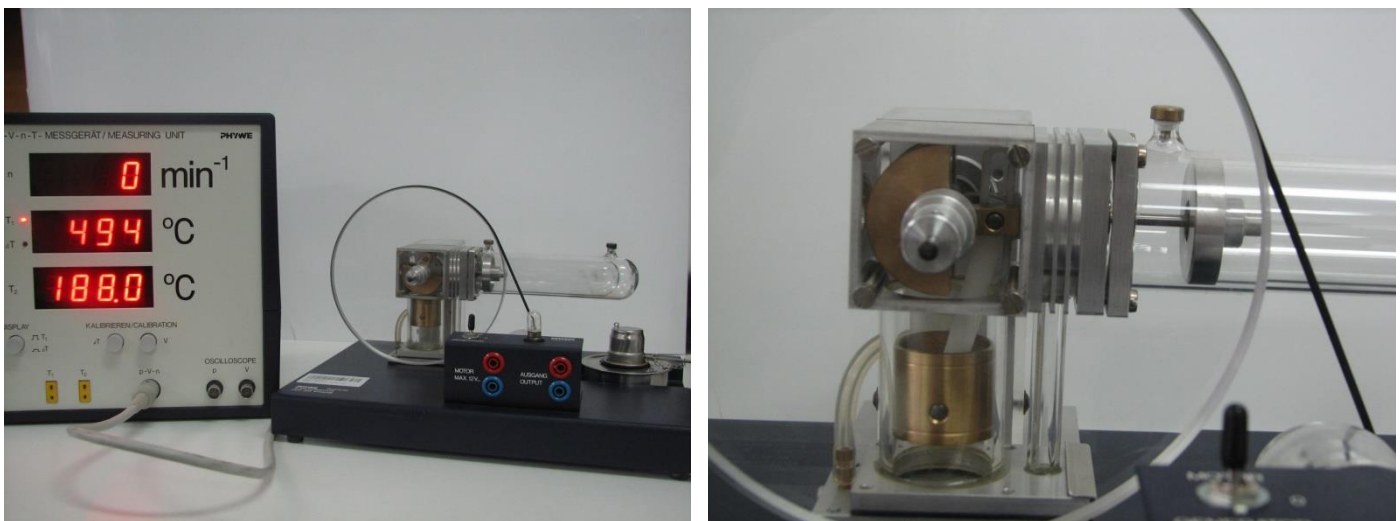


Fig. 1. View of Stirling engine manufactured by PHYWE

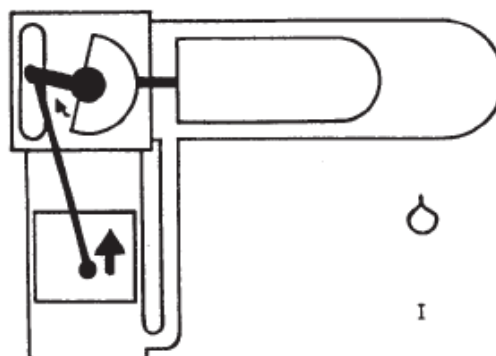
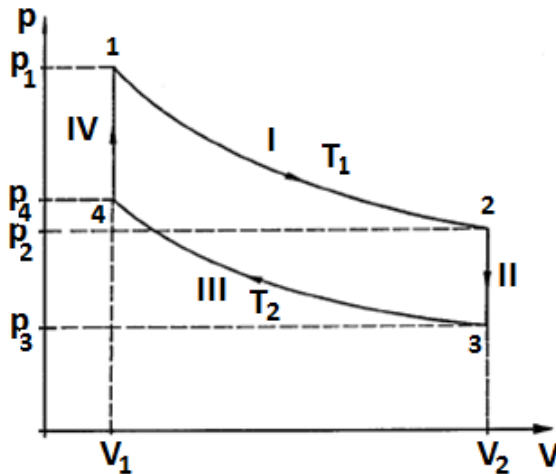
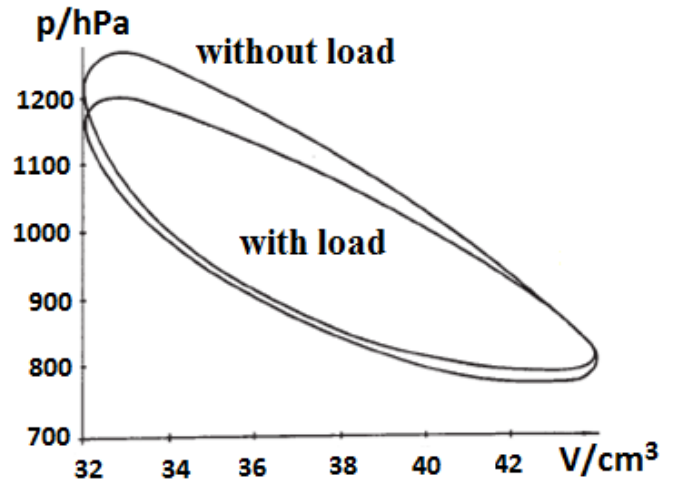


Fig. 2. Scheme of Stirling engine visible in Fig. 1

In the extension of the axis of rotation cylindrical shield made of plexiglass with a rubber strap connected to an electrical generator and counter rotation is fixed. Additionally, on the outer surface of the warm cylinder (Fig. 1) there are the mounting points of the hot end of the sheathed K-type thermocouple T1 and T2 for measuring temperature. The theoretical and the real cycle of Stirling engine are shown in Fig. 3.



Rys. 3a. Theoretical Stirling cycle



Rys. 3b. Real Stirling cycle

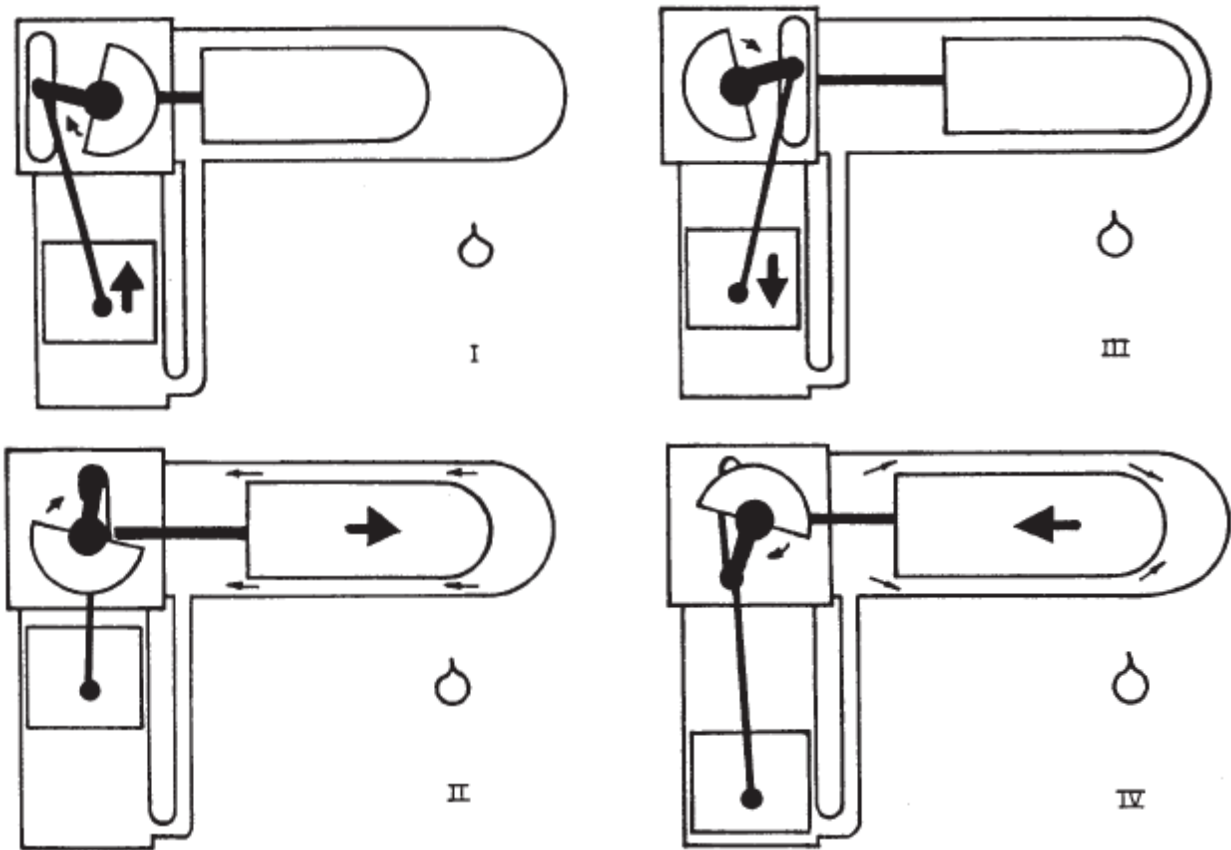
The idealised Stirling cycle consists of four thermodynamic processes acting on the working fluid (see Fig. 3a):

- I. Isothermal expansion ( $T_1 = \text{idem}$ ). The expansion-space and associated heat exchanger are maintained at a constant high temperature, and the gas undergoes near-isothermal expansion absorbing heat from the hot source.
- II. Isochoric heat-removal ( $V_2 = \text{idem}$ ). The gas is passed through the regenerator (heat radiator) where it cools, transferring heat to the regenerator for use in the next cycle.
- III. Isothermal compression ( $T_2 = \text{idem}$ ). The compression space and associated heat exchanger are maintained at a constant low temperature so the gas undergoes near-isothermal compression rejecting heat to the cold sink
- IV. Isochoric heat-addition ( $V_1 = \text{idem}$ ). The gas passes back through the regenerator where it recovers much of the heat transferred in II, heating up on its way to the expansion space.

According to the first law of thermodynamics, the heat  $dQ$  supplied from the outside of a closed system is used up on the stationary growth of its internal energy  $dU$  and external work  $dL$  which is equal to the change in volume of work  $dL = pdV$

$$dQ = dU + pdV \quad (1)$$

While operating the Stirling engine heats and cools the working fluid in a cyclic manner gaining energy during its expansion and compression



Rys. 4. Subsequent phases of the Stirling cycle (cf. Fig. 3a)

*Remark.* Work of isothermal process takes place without a change of internal energy ( $dU=0$ ) of the system.  
From Eq.1 we have  $dQ_{T1-2} = pdV$

Phase I: The heated gas in the warm cylinder (horizontal cylinder, a piston leaks) expands and passes to the cold cylinder (vertical cylinder, sealed piston) displacing it upward. At the time the isothermal work  $L_{T,I}$  is done

$$L_{T,I} = -n\bar{R}T_1 \ln(V_2/V_1) = -n\bar{R}T_1 \ln(p_1/p_2), \quad \bar{R} = 8314.3 \text{ J mol}^{-1}\text{K}^{-1} \quad (2)$$

where  $n$  – number of gas moles contained in the system, the minus sign (—) means that the work is supplied to the system

Phase II: The piston in the cold cylinder remains almost at rest, because it is in its upper turning position. Gas fills almost the entire volume of the cold cylinder and begins to cool. This causes a drop in pressure close to isochoric conditions. During the isochoric transition the amount of heat dissipated from the system is

$$Q_{v,II} = n\bar{C}_v(T_2 - T_1) = n \frac{\bar{R}}{\kappa - 1} (T_2 - T_1) \quad (3)$$

where  $\kappa = c_p/c_v$  is the isentropic exponent (for air  $\kappa = 1.4$ )

Phase III: The movement of the flywheel and contraction of the gas causes the cold piston down, wherein, the warm piston is in its right-turning position (nearly stationary) which limit the flow of heat from the

burner. Hot and cold gases (under the influence of movement of the piston cold) are mixed together and dissipate heat through the heat sink to the surroundings and in addition to this isothermal work  $L_{T,III}$  is done

$$L_{T,III} = +n\bar{R}T_2 \ln(V_2/V_1) = n\bar{R}T_2 \ln(p_1/p_2) \quad (4)$$

where the following relation takes place  $L_{T,III} < |L_{T,I}|$  because  $T_2 < T_1$

Phase IV: Piston in cold cylinder reaches its lower position feedback, and the moving piston in a warm cylinder causes compression of the gas left in almost isochoric conditions, and therefore the gas temperature rise due to the heat release and the system returns to the phase I. The amount of heat evolved during the isochoric process IV is the heat supplied to the system of the value

$$Q_{v,IV} = n\bar{C}_v(T_1 - T_2) = n\frac{\bar{R}}{\kappa - 1}(T_1 - T_2) \quad (5)$$

The total amount of heat supplied to the system is equal to

$$Q_d = Q_{T,I} + Q_{v,IV} = n\bar{R}T_1 \ln(V_2/V_1) + n\bar{R}\frac{1}{\kappa - 1}(T_1 - T_2) \quad (6)$$

and dissipated from the system

$$Q_{od} = Q_{T,III} + Q_{v,II} = -n\bar{R}T_2 \ln(V_2/V_1) - n\bar{R}\frac{1}{\kappa - 1}(T_1 - T_2) \quad (7)$$

According to definition of the thermal efficiency of heat engine we have

$$\begin{aligned} \eta_{ts} &= \frac{L_{ob}}{Q_d} = \frac{Q_d - |Q_{od}|}{Q_d} = 1 - \frac{|Q_{od}|}{Q_d} = 1 - \frac{n\bar{R}\left[T_2 \ln(V_2/V_1) + \frac{1}{\kappa - 1}(T_1 - T_2)\right]}{n\bar{R}\left[T_1 \ln(V_2/V_1) + \frac{1}{\kappa - 1}(T_1 - T_2)\right]} = \\ &= 1 - \frac{T_2[(\kappa - 1)\ln(V_2/V_1) + (T_1/T_2 - 1)]}{T_1[(\kappa - 1)\ln(V_2/V_1) + (1 - T_2/T_1)]} < 1 - \frac{T_2}{T_1} = \eta_{tc} \end{aligned} \quad (8)$$

and therefore the Stirling cycle thermal efficiency is less than the thermal efficiency of the Carnot cycle. In the case of Stirling cycle one should bear in mind that only a portion of the total work  $L_{ob}$  is done efficiently at a given load and additionally there is an irreversible loss of energy which is consumed to overcome friction.

### 3. EXPERIMENTAL

The laboratory stand (Fig. 5) consists of a Stirling engine (Fig. 1) and the measuring unit purchased from PHYWE. The laboratory stand also includes equipment associated with the data acquisition module from National Instruments.

LabVIEW software along with a program that supports the operation of the laboratory to study the Stirling engine is used. As a preliminary point the NI 9205 measurement card (card for voltage measurement) and NI 9211 card (for temperature measurement) module NI cDAQ-9172 must be installed (see Fig. 6)

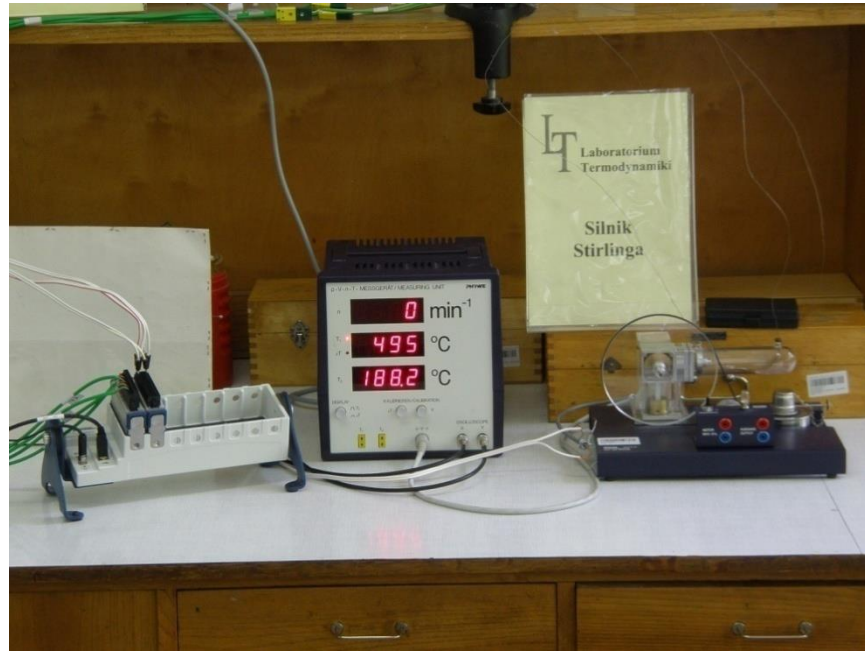


Fig. 5. Laboratory stand for Stirling engine investigation.

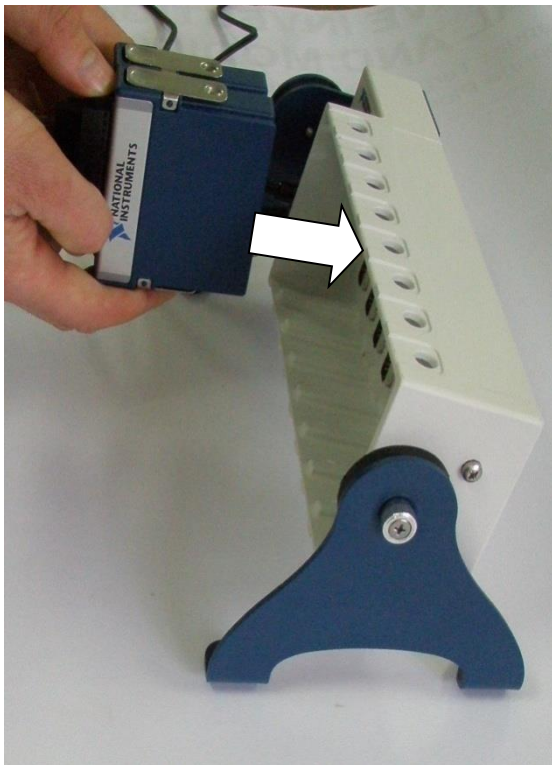


Fig. 6. Installation of measurement cards NI 9205 and NI9211 in module NI cDAQ-9172



Fig. 7 NI 9205 Card slots with channel numbering



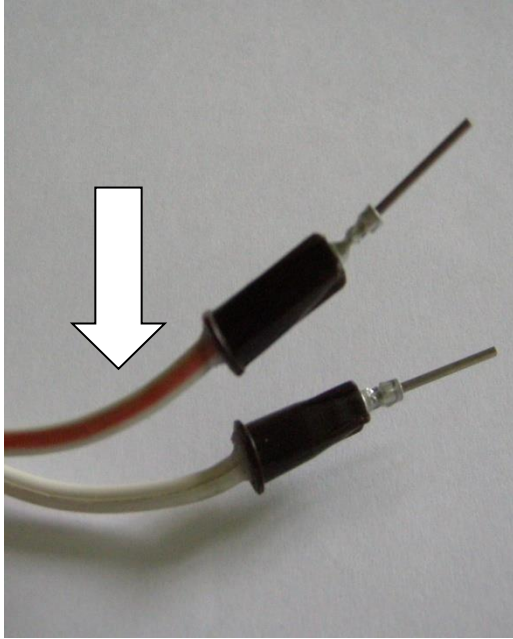


Fig. 8 Cable with a visible pink strap intended for the "+"

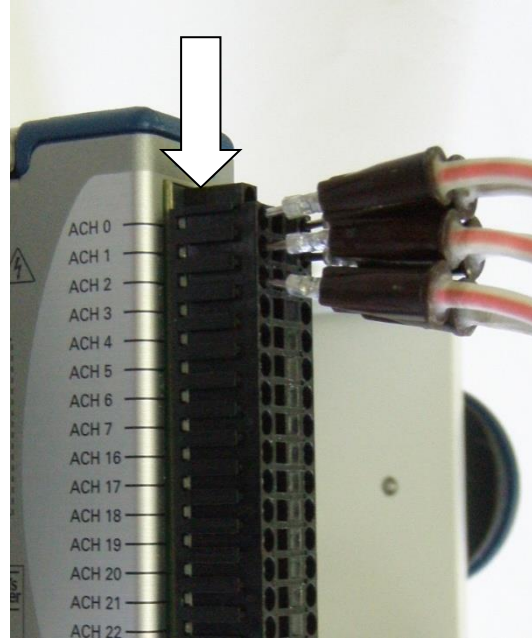


Fig. 9. Card slot NI 9205 with inserted pin cables.

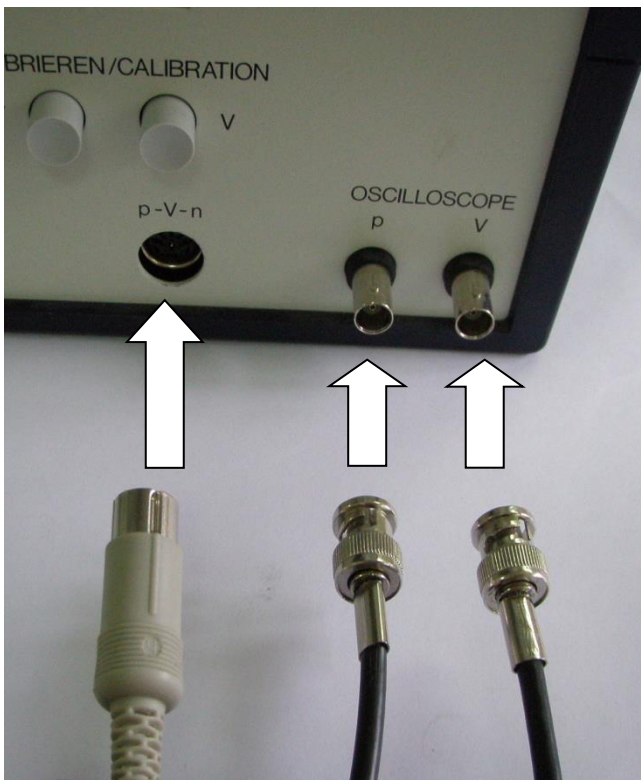


Fig. 10. Cables with connectors type oscilloscope and 8-pin DIN plug( left),

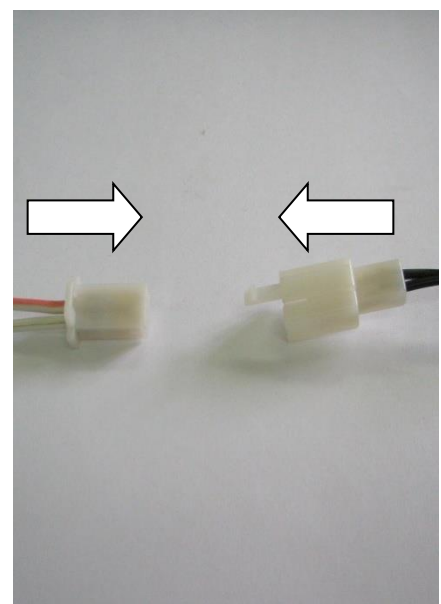


Fig. 11. Plug and socket as a quick-connector.

#### 4. PREPARATION OF THE LABORATORY TEST BENCH FOR MEASUREMENTS

The test bench (Fig. 5) consists of a Stirling engine model (Fig. 1) and a measurement module purchased from PHYWE. The test bench is also equipped with a data acquisition module from National Instruments (Ni 9172 - 8-slot, NI 9211 thermocouple card and NI 9205 voltage card).

The test stand is also equipped with a PC with installed LabVIEW environment and a program supporting the operation of the laboratory test stand for Stirling engine tests.

The connections of the test bench modules must be verified:

- connection of the Stirling engine module with the Phywe measuring module and the NI 9205 measuring card,
- the connection of the Stirling engine model thermocouples to the Phywe measuring module and the NI 9211 measuring card,
- check the belt from the flywheel to the motor/generator,
- set the lower piston of the Stirling engine to the minimum position (lowest piston position).
- Check the connection of the NI cDAQ-9172 module to the power supply and also via the USB cable to the PC. Switch on the module using the button located on its front panel,
- switch on the PC power supply and start the Stirling engine application (special icon on the monitor screen),
- switch on the Phywe measuring module and calibrate it at the lowest piston position of the Stirling engine model. Press the buttons marked " $\Delta T$ " and "V" on the front panel of the measuring block under "CALIBRIEREN/CALIBRIATION". The volume sensor is automatically calibrated and the temperature and rotation speed values appear on the display of the measuring block.

Refill the metal torch reservoir with pure ethanol. Using the knob on the housing, extend the wick to a length of 7 [mm] (Fig. 7), measure its mass  $m_{p1}$  using a laboratory balance. This allows the subsequent determination of the change in weight of the burner  $A_m$ , and thus the change in weight of the spirit

#### 5. MEASUREMENTS

Start the **Stirling engine** program - icon on the PC screen.

The starting view of the measuring instrument, developed in LabView, is shown in Figure 12.

Several fields/buttons important for controlling the operation of the instrument can be distinguished.

- A. A line of tool buttons with buttons to start and stop the application.
- B. The "STOP" button, which allows you to switch off a running program, including recording to a file if previously activated. The green LED (beside) confirms that the programme is running.
- C. The speed  $n$  in the "Panel" window is displayed by means of a pointer on the dial indicator and additionally below it in digital form. This speed is given in the unit [rpm].
- E. Instantaneous maximum and minimum values of pressure  $p$  and gas volume  $V$ .
- F. The linear digital indicators show the temperature values at the four measuring points and the absolute value of the difference between the temperature  $T_H$  of the hot part of the cylinder and the temperature  $T_K$  of the cold part of the cylinder.



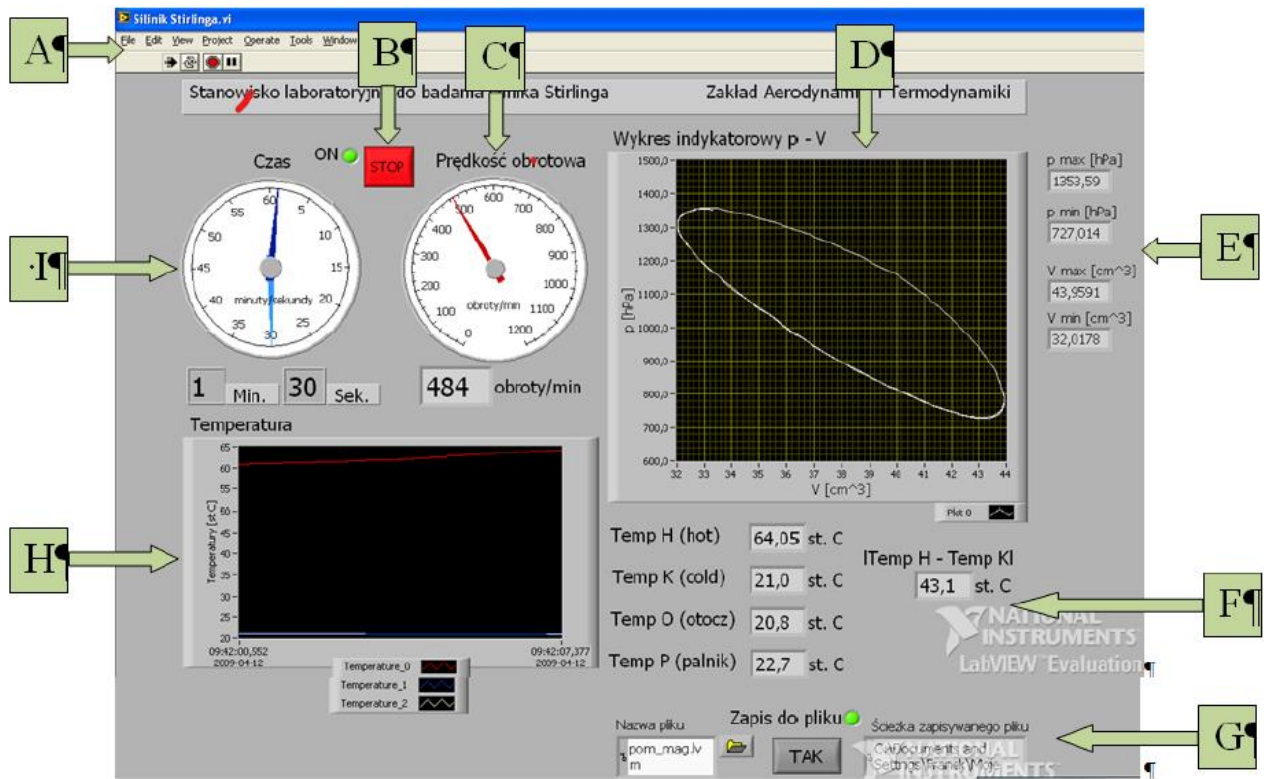


Fig. 12. The "panel" running program with the indications of individual elements

G. Button for switching on or off the possibility of saving to a file during program operation. This button is responsible for saving all the measured values to a text file with the extension .txt, lvm or tdm. Configuration of parameters allows selection of the storage path and the name under which the file is to be saved. It is also possible to see the name of the file being saved at any given time in the adjacent window. A green LED next to the display confirms that the file is being saved.

We use the mouse to operate the programme.

We can now enter (field G) the file name under which the file is to be saved, define the extension of the text file (.txt, .lvm or .tdm) and choose the path to save it on the computer's hard disk. If you do not enter a customised file name, extension and path, the file will automatically be saved on computer hard disk under the name "test" with the extension ".lvm" in the "LabVIEW Data" directory located in the "My Documents" folder.

Light the wick of the burner and position the burner under the outer end of the horizontal glass cylinder. At the same time switch on the program by pressing the white roller in the A field. (the LED with the word "ON" should light green). After approx. 1- 2 minutes, try to rotate the motor disc clockwise to initiate the Stirling motor. When the motor starts running, start recording the measurement data to a file (button G) located in the bottom right corner of the screen.

The duration of the measurements is between 10 and 15 minutes, depending on the recommendation of the practitioner. The values are measured and recorded with a step of approx. 3 seconds. The measurement results in a text file with the measured values in columns, one row for each individual measurement. Table 1 gives the column number, name and unit of the quantity stored in the result file.

Table 1

Assignment of the measured quantity to a column number in the result file

Column number	Measured quantity	Unit
1	Time t	s

2	Volume $V$	$\text{cm}^3$
3	Pressure $p$	hPa
4	Maksimum pressure $p_{\text{MAX}}$	hPa
5	Minimum pressure $p_{\text{MIN}}$	hPa
6	Engine speed $n$	obr / min
7	Temperature of the "hot" part of the cylinder $T_H$	$^{\circ}\text{C}$
8	Temperature of the "cold" part of the cylinder $T_K$	$^{\circ}\text{C}$
9	Ambient temperature $T_{\text{OT}}$	$^{\circ}\text{C}$
10	Brak przypisania	-----

### **Measurements and elaboration of results**

The external heat source is a burner in which pure ethyl alcohol is burned.

Considering the parameters of the burner:

- container volume  $\Delta V = 29 \text{ ml}$ ,
- alcohol density  $\rho = 0.83 \text{ g/ml}$ ,
- specific heat output  $h = 25 \text{ kJ/g}$ ,
- combustion time  $\Delta t = 3600 \text{ s}$ .

Unit (mass) alcohol consumption can be determined

$$\frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{0,83 \cdot 29}{3600} = 6,686 \cdot 10^{-3} \text{ [g s}^{-1}\text{]} \quad (9)$$

and the amount of heat per second (thermal power) released during burner operation

$$P_H = \frac{\Delta m}{\Delta t} \cdot h = 6,686 \cdot 10^{-3} \cdot 25000 \approx 167 \text{ W}$$

- a) Using the data in the result file, determine the time  $t$  of measurement.
- b) Determine the mass loss  $\Delta m_p$  of the burner during the combustion (at time  $t$ ) of the spirit contained in it, knowing the initial mass of the burner  $m_{p1}$ , from the relation:

$$\Delta m_p = -0,008 \cdot t + m_{p1} \quad (10)$$

- c) Calculate the arithmetic mean values of the "hot"  $\overline{T_H}$  and "cold"  $\overline{T_K}$  temperatures of the cylinder parts of the engine model, recorded in the result set. Use the expression (11)

$$\overline{T} = \frac{\sum_{i=1}^n T_i}{n} \quad (11)$$

- (d) Determine the temperature coefficient  $E$  for the calculated average temperatures  $\overline{T_H}$  and  $\overline{T_K}$

$$\bar{\tau} = \frac{\overline{T_K}}{\overline{T_H}} \quad (12)$$

- e) Determine the maximum  $V_{max}$ , and minimum  $V_{min}$  volume obtained during the measurements  
 (f) Determine the compression ratio for the above values of  $V_{max}$  and  $V_{min}$  from the formula:

$$\varepsilon = \frac{V_{max}}{V_{min}} \quad (13)$$

(Note: Assume  $V_{min} = 32 \text{ cm}^3$ ,  $V_{max} = 44 \text{ cm}^3$ ,  $\Delta V = 12 \text{ cm}^3$ . The number of moles of air contained in the engine cylinders is  $n_m = 1.10 \cdot 10^{-3} \text{ mole}$ )

- (g) Calculate the thermal efficiency  $\eta_t$  of the Stirling engine cycle (assuming  $\kappa = 1.4$ ) from the formula:

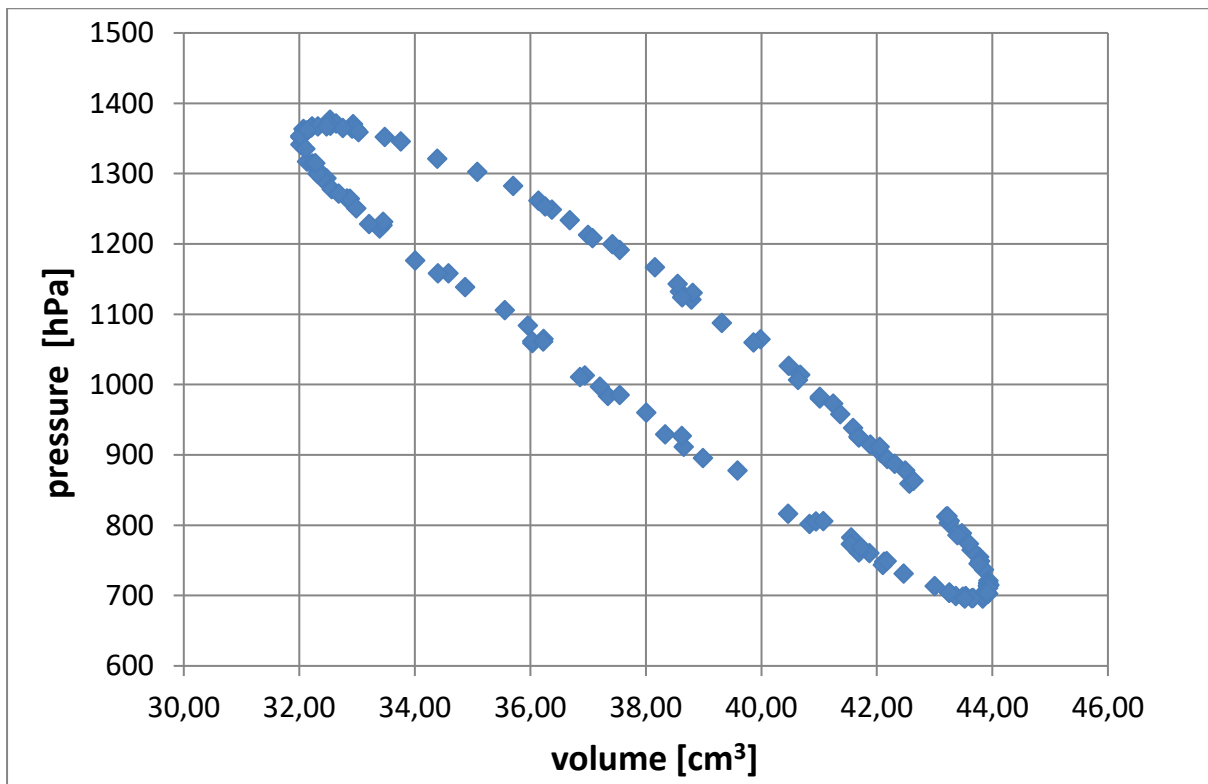
$$\eta_t = \frac{(\kappa - 1) (1 - \bar{\tau}) \ln \varepsilon}{(\kappa - 1) \ln \varepsilon + (1 - \bar{\tau})} \quad (14)$$

- (h) Calculate the thermal efficiency  $\eta_t$  for the Carnot cycle (in the same temperature range  $T_{min} = \bar{T}_K$  and  $T_{max} = \bar{T}_H$ , from the formula:

$$\eta_{tC} = 1 - \frac{\bar{T}_K}{\bar{T}_H} = 1 - \bar{\tau} \quad (15)$$

Make graphs:

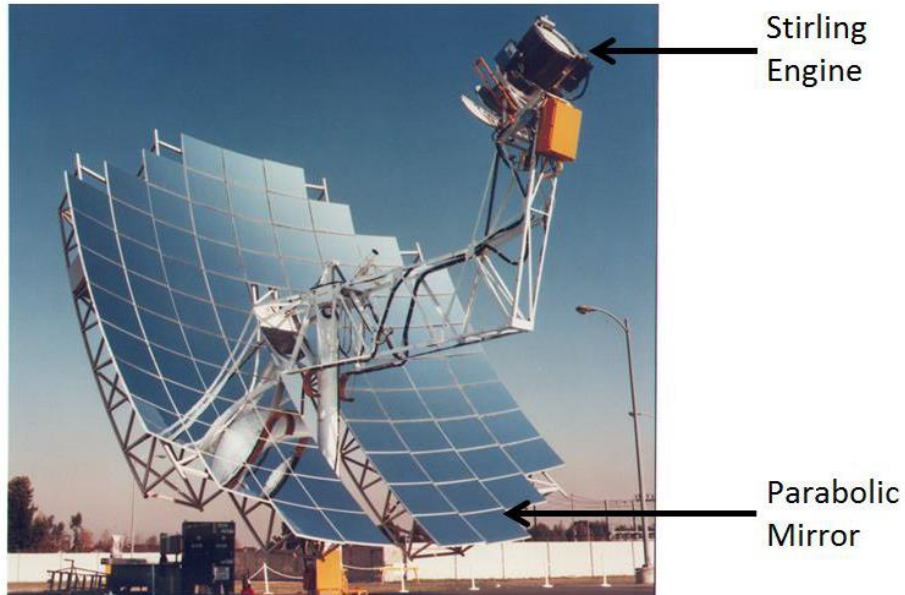
- The curves of the measured values of temperature  $T_K$ ,  $T_H$ ,  $T_O$  and the curve of the change in speed  $n$  as a function of time  $t$ . Plot these curves on a single graph in Excel (two independent y-axes of this graph).
- For pressure  $p$  as a function of volume  $V$  - point diagram  $p - V$  in Excel. An example of a  $p-V$  chart is shown in the figure below.



6. APPLICATIONS OF STIRLING ENGINE [5]

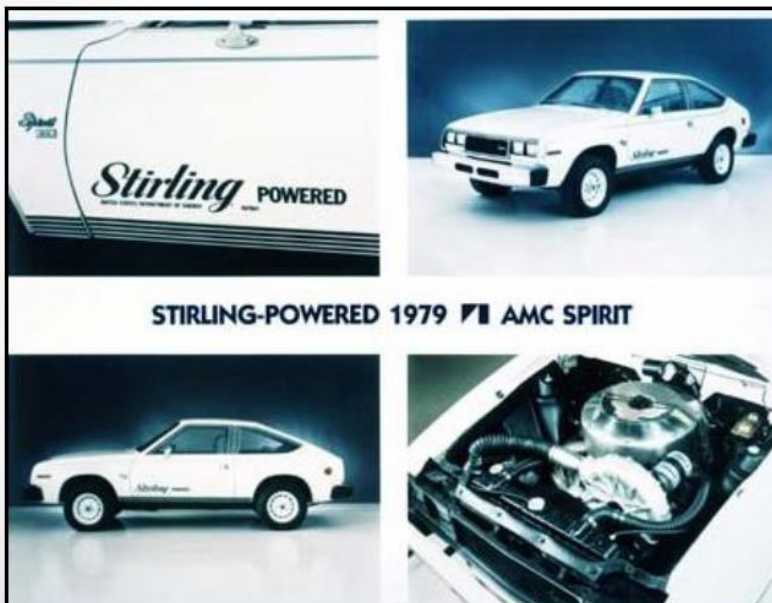
## Solar Power Generation

Stirling solar units are capable of generating 25kW electricity



## Automobiles using Stirling engines

Research led by General Motors and Ford – not a success



Low power to weight ratio

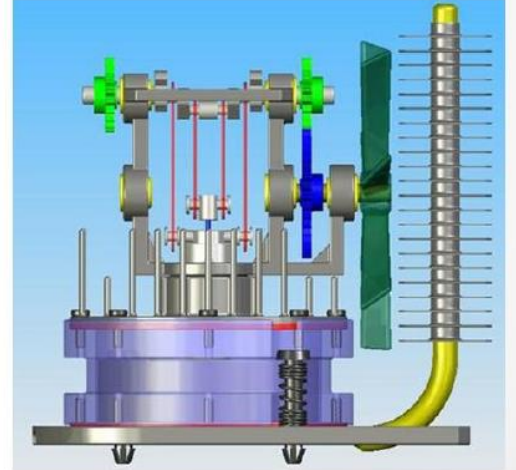
More expensive than internal combustion engines for same power output.

Require a longer warm up time



## Computer chip cooling

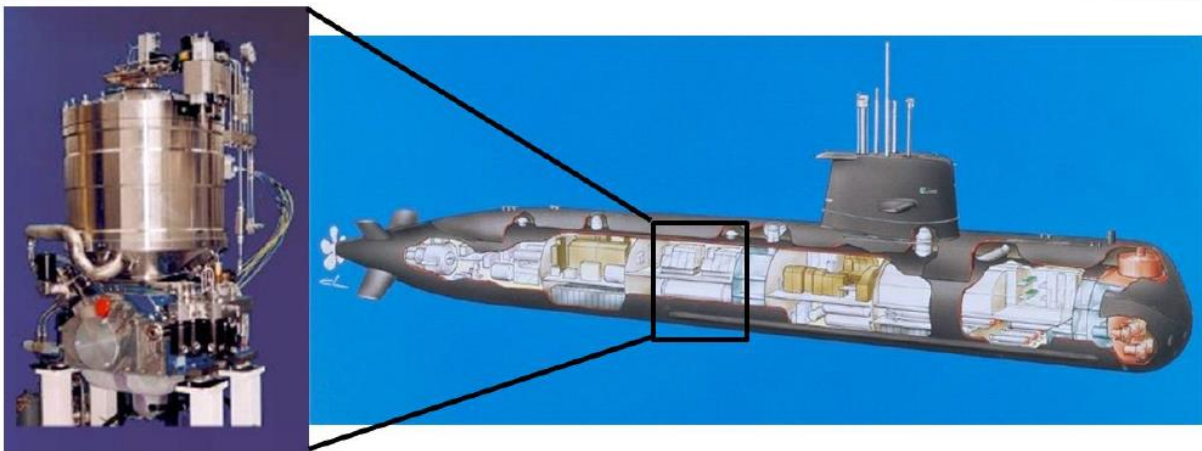
Micro-Star International Co., Ltd, Taiwan. Miniature Stirling engine. Uses heat from the processor to cool the processor



MSI claim the engine is 70 per cent efficient

## Stirling engine powered submarines

Remarkably quiet – backup to primary modern diesel-electric engines when a silent approach is required

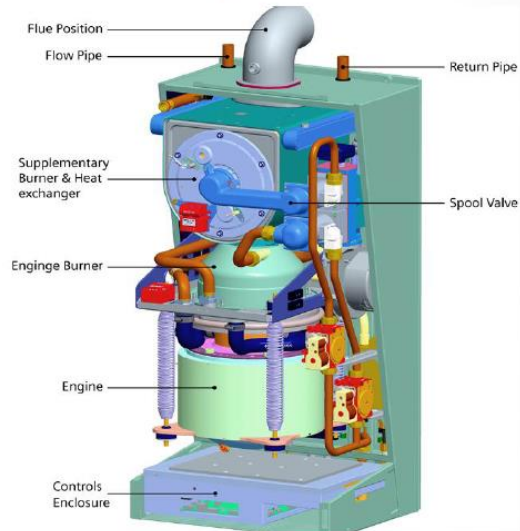


Stirling engines are used to power a 75kW generator



## Domestic heat and power - Stirling CHP unit

Fuel drives Stirling engine to generate mechanical power to produce electricity. Waste heat from engine heats home.



## Low temp. difference type Stirling engine



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